

# Sino-French Summer Institute: Stochastic Modeling and Applications

Sino-French Summer Institute 2011 in Stochastic Modeling and Applications (CNRS-NSFC Joint Institute of Mathematics), will be held at Academy of Mathematics and System Science, Beijing, on June 13rd–July 1st, 2011.

This summer institute is organized by Institute of Applied Mathematics, AMSS, Key Laboratory of Random Complex Structures and Data Science, CAS, and Laboratoire de Probabilités et Modèles Aléatoires, Université Pierre et Marie Curie - Paris VI et Université Paris Diderot - Paris VII.

The themes of the summer institute are around the stochastic modeling: theory and applications. We are going to organize several mini-courses in the first two weeks (June 13th - June 26th) and a workshop in the last week (June 27th - July 1st). We focus mainly on the following topics.

1. Numerical probabilities and related topics
2. New approaches in stochastic analysis
3. Financial mathematics and models

## Abstract of Mini-course

### **On the concentration properties of mean field particle models**

(based on a series of 4 joint works with : D.A. Dawson, A. Guionnet, E. Rio, S.L. Hu and L.M. Wu)

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#### **Abstract**

This lecture is concerned with the exponential concentration properties of a general class of mean field particle interpretations of nonlinear measure valued processes. We discuss large and moderate functional deviations principles w.r.t weak and strong topologies, as well as non asymptotic exponential concentration inequalities.

In the first part, we underline three generic strategies to obtain a large-deviation principle, namely Cramer's method, Laplace-Varadhan integral techniques, and Dawson-Gartner projective limit techniques.

The second part of the lecture is concerned with moderate deviations principles. Our approach is based on a original semigroup analysis combined with stochastic perturbation techniques and Dawson-Gartner type projective limit deviation methods.

The non asymptotic concentration inequalities presented in the third part of this lecture generalize the classical Hoeffding, Bernstein and Bennett inequalities for conventional independent random sequences to interacting particle systems. We combine an original stochastic perturbation analysis with a concentration analysis for triangular arrays of conditionally independent random sequences.

Under some additional stability properties of the limiting measure valued processes, uniform concentration properties with respect to the time parameter are also derived. We also illustrate these exponential concentration properties in the context of McKean Vlasov type diffusion models, McKean collision type models of gases, and of a class of Feynman-Kac distribution flows arising in engineering sciences, Bayesian statistics, as well as in biology and in molecular chemistry.

**References :**

[1] P. Del Moral, S.L. Hu & L.M. Wu Moderate Deviations for Mean Field Particle Models. Preprint (2011).

[2] Pierre Del Moral, Rio Emmanuel. Concentration Inequalities for Mean Field Particle Models HAL-INRIA RR-6901 to appear in the Annals of Applied Probability (2011).

[3] Dawson D.A. and Del Moral P. Large deviations for interacting processes in the strong topology. Statistical Modeling and Analysis for Complex Data Problem P. Duchesne and B. Rmillard Editors, pp. 179–209, Springer (2005).

[4] Del Moral P., Guionnet A. Large Deviations for Interacting Particle Systems. Applications to Non Linear Filtering Problems. Stochastic Processes and their Applications, vol. 78, pp. 69-95 (1998).

## Fractional regularity: applications in stochastic

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### Abstract

In this lecture, we will introduce and study the concepts of fractional regularity of random variables or stochastic processes. These notions play a central role in the approximation of stochastic integrals, in the derivation of tight estimates on PDE having probabilistic representations, in discretizing Backward Stochastic Differential Equations... We will review most recent results.

# Enlargement of filtrations and applications to credit risks

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## Abstract

The theory of enlargement of filtrations has been developed by the French school of probability during the 70-80s. In this course, we will introduce two types : the initial enlargement and the progressive of enlargement of filtrations and discuss some fundamental problems such as the decomposition of martingales, immersion property, etc.. The last part of the course will be dedicated to the applications to finance, notably to the modeling of credit risks.

# Introduction of Backward Stochastic Differential Equations

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*Université du Maine, France*

## Abstract

We will begin with BSDE's with lipschitz coefficient, to study the Existence and unicity of the solution, and the comparison theorem, with the extension to the linear increasing case, and to the superlinear quadratic increasing case. Then we study BSDE's with one reflected barrier for the Lipschitz case, with its Extensions. At last we will consider application to zero-sum stochastic differential games.

### REFERENCES.

- [1] El Karoui, Peng, Quenez: Backward stochastic differential equations in Finance.Math Finance 7 (1997)
- [2] Lepeltier, San Martin: BSDE's with continuous coefficient-Stat. and Prob. Letters 32(1997)
- [3] Lepeltier, San Martin: Existence for BSDE's with superlinear quadratic coefficient-Stoch. and Stoch. Report vol.63(1998)
- [4] El Karoui, Kapoudjian, Pardoux, Peng, Quenez: Reflected solutions of BSDE's and related obstacle problem for PDE's, Ann. Probab. 25, no 2, 702-737 (1997).
- [5] Matoussi: Reflected solution of BSDE's with continuous coefficient.Stat. and Prob. Letters 34 (1997)
- [6] Kobilanski, Lepeltier, Quenez, Torres: Reflected BSDE with superlinear quadratic coefficient.Prob. and Math. Statistic,Vol.22,Fas.1(2002)
- [7] Hamadene, Lepeltier: Zero-sum stochastic differential games and Backward equation.System and Contol Letters 24 (1995)
- [8] Hamadene-Lepeltier:Reflected BSDE's and mixed game problem.SPA 85 (2000)

# Introduction to stochastic approximation with application to finance

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Université Paris VI, France*

## Abstract

Recursive stochastic algorithms are simulation based zero search procedures which can be seen as a Monte Carlo approach to the solving of inverse and optimization problems. We will provide some classical background (supermartingale method, ODE, SDE methods) about the asymptotic behavior of such procedures (a.s. convergence, rate, ...) As a second step, we will show how we appear naturally in various fields of finance like calibration, risk management, asset allocation, etc..

## A New Approach in Backward Stochastic Differential Equation

*Zhongming Qian*

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## Abstract

We study BSDEs in on a general filtered probability space, which does not depend on any martingale representation, and thus allows to study a wide class of backward stochastic dynamics. Under certain assumptions, such BSDE admits a unique solution. In this way, such backward stochastic dynamics, as a class of Markov processes, can be regarded as a generic extension of some non-linear PDE problems of finite dimension to infinite dimensional problems in path spaces.

## An Introduction to $G$ -expectaion

*Yongsheng Song*

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## Abstract

This mini course is focused on the basic notions and recent developments on  $G$ -expectation, which is introduced by Prof. Shige Peng.

# Pricing and risk management with Lévy processes

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*Ecole Polytechnique, France*

## Abstract

The recent period of extreme volatility in financial markets has once more drawn the attention of academics and practitioners to the insufficiency of Gaussian modelling and the importance of taking into account the extreme market moves. The aim of this course is to show that Lvy processes now offer an easy to use toolkit for pricing and hedging the jump risk in financial markets. After a brief overview of the mathematical aspects of Lvy processes, we concentrate on their uses in risk management, exploring the financial applications where using jump processes really makes a difference.

# Second order Backward Stochastic Differential Equation

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*Ecole Polytechnique, France Department of Mathematics,*  
*University of Southern California, USA*

## Abstract

The second order BSDE, in its Markovian case, provides a probabilistic representation for fully nonlinear PDEs and thus opens the door for Monte Carlo methods for high dimensional fully nonlinear PDEs. The theory has various applications in finance, in particular on models with volatility uncertainty. This theory is also closely related to the  $G$ -expectation, a nonlinear expectation proposed by Peng. In this mini course we will introduce the basics of the theory as well as some applications.

# Large Deviation

*Liming Wu*  
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## Abstract

We introduce the framework of large deviation, basic results and techniques, together with some numerical applications. Principles of large deviations may be effectively applied to gather information out of a probabilistic model. Different phenomena such as those in statistics, random perturbation of dynamical systems, large time behavior of Markov processes will be studied by using large deviations.