Abstracts of talks in Workshop on Stochastic Modelling and Applications to Finance

1. Mohamed Ben Alaya (Université Paris 13)

   Asymptotic Behavior of The Maximum Likelihood Estimator For Ergodic and Nonergodic Sqaure-Root Diffusions

   In this talk, based on joint work with A. Kebaier, we will discuss the problem of parameter estimation in the Cox-Ingersoll-Ross (CIR) model $(X_t)_{t\geq 0}$. This model is frequently used in finance for example to model the evolution of short-term interest rates or as a dynamic of the volatility in the Heston model. We establish asymptotic results on the maximum likelihood estimator (MLE) associated to the drift parameters of $(X_t)_{t\geq 0}$. To do so, we need to study first the asymptotic behavior of the quadruplet $(\log X_t, X_t, \int_0^t X_s ds, \int_0^t \frac{ds}{X_s})$. This allows us to obtain various and original limit theorems on our MLE, with different rates and different types of limit distributions. Our results are obtained for both cases : ergodic and nonergodic diffusion.

2. Nan Chen (Chinese university of Hong Kong)

   A Non-Zero-Sum Game Approach for Convertible Bonds : Tax Benefits, Bankrupt Cost and Early/Late Call

   Convertible bonds are hybrid securities that embody the characteristics of both straight bonds and equities. The conflict of interests between bondholders and shareholders affects the security prices significantly. In this paper, we investigate how to use a non-zero-sum game framework to model the interaction between bondholders and shareholders and to evaluate the bond accordingly. Mathematically, this problem can be reduced to a system of variational inequalities and we explicitly derive the Nash equilibrium to the game. Our model shows that credit risk and tax benefit have considerable impacts on the optimal strategies of both parties. The shareholder may issue a call when the debt is in-the-money or out-of-the-money. This is consistent with the empirical findings of "late and early calls" (Ingersoll (1977), Mikkelson (1981), Cowan et al. (1993) and Asquith (1995)). In addition, the optimal call policy under our model offers an explanation for certain stylized patterns related to the returns of company assets and stock on calls.

3. Rama Cont (Université Paris VI)

   TBA

4. Areski Cousin (Université de Lyon I)

   Modeling dynamic portfolio credit risk with common shocks

We consider a bottom-up Markovian portfolio credit risk model where dependence among default times stems from the possibility of simultaneous defaults. A common shocks interpretation of the model is possible so that efficient convolution recursion procedures are available for pricing and hedging CDO tranches, conditionally on any given state of the Markov model. Calibration of marginals and dependence parameters can be performed separately using a two-steps procedure, much like in a standard static copula set-up. As a result this model allows us to hedge CDO tranches using single-name CDS-s in a theoretically sound and practically convenient way. To illustrate this we calibrate the model against market data on CDO tranches and the underlying single-name CDS-s. After the calibration, which renders good fits, we study the implied loss distributions as well as the implied min-variance hedging strategies in the calibrated portfolios.

5. Stéphane Crépey (Université d'Evry)

   Counterparty Risk on Interest Rate Derivatives in a Multiple Curve Setup

   This is a joint work with Zorana Grbac. We study the valuation and hedging of CSA interest rate derivatives. By CSA interest rate derivatives, we mean a portfolio of OTC interest rate derivatives between two defaultable counterparties, connected by the means of a legal agreement called a Credit Support Annex (CSA) regarding the counterparty risk related cash-flows. CSA cash-flows comprise the collateral related to this portfolio, and the close-out cash-flows in case of default of either party.

   The first step consists in the so-called counterparty clean valuation of the portfolio, namely the valuation in a hypothetical situation where the parties would be risk-free, yet accounting for the post-crisis discrepancy between the risk-free discount curve, and the LIBOR fixing curve. Toward this end we resort to a defaultable HJM methodology, in which this discrepancy is accounted for by the possibility of a stylized default of the LIBOR contributing banks. Parsimonious short rate specifications are given in the form of an extended CIR and a Lvy Hull–White model for the risk-free short rate and the LIBOR short credit spread.

   In the second step, the counterparty clean value process of the portfolio is used as an underlier to an option called Contingent Credit Default Swap (CCDS), which prices the correction in value known as the Credit Valuation Adjustment (CVA) to the portfolio due to the counterparty risk. The post-crisis multiple curve issue (including the above discrepancy) implies that the CVA should also account for specific costs of funding a position in the portfolio and in its collateral, and of setting-up a related hedge. We model funding costs in the form

of credit and liquidity bases. We develop a reduced-form backward stochastic differential equations (BSDE) approach to the problem of pricing and hedging interest rate counterparty risk. In the simplest cases this problem can be reduced to low-dimensional Markovian pre-default CVA BSDEs, or equivalent semi-linear PDEs.

6. Min Dai (National University of Singapore)
   Optimal Consumption and Investment with Differential Long-term and Short-term Tax Rates
   We propose a novel optimal consumption and optimal investment model with differential long-term/short-term tax rates. We formulated it as a stochastic control problem and the associated value function satisfies a quasi-variational inequality equation. The optimal trading strategy is characterized by a time-varying no-transaction region outside which it is optimal to sell the stock and repurchase it to achieve an optimal fraction of after-tax wealth invested in stock. In contrast to the standard literature, we show that it can be optimal to defer capital loss even in the absence of transaction costs. In addition, a small investor subject to capital gain tax may be better off than a tax-exempt small investor, and a small investor may prefer a taxable security to a tax-exempt security. Moreover, raising the short-term tax rate can increase both consumption and stock investment. This work is joint with Hong Liu and Yifei Zhong.
7. Sylvain Delattre (Université Paris 7)
   Modeling microstructure noise with mutually exciting point processes
   This is a joint work with E. Bacry, M. Hoffmann and J.F Muzy. We introduce a new stochastic model for the variations of asset prices at the tick-by-tick level in dimension 1 (for a single asset) and 2 (for a pair of assets). The construction is based on marked point processes and relies on linear self and mutually exciting stochastic intensities as introduced by Hawkes. We associate a counting process with the positive and negative jumps of an asset price. By coupling suitably the stochastic intensities of upward and downward changes of prices for several assets simultaneously, we can reproduce microstructure noise (i.e. strong microscopic mean reversion at the level of seconds to a few minutes) and the Epps effect (i.e. the decorrelation of the increments in microscopic scales) while preserving a standard Brownian diffusion behaviour on large scales.
8. Pierre Del Moral (INRIA)
   A Backward Particle Interpretation of Feynman-Kac Formulae
   This is a joint work with A. Doucet and S.S. Singh. We design a particle interpretation of Feynman-Kac measures on path spaces based

on a backward Markovian representation combined with a traditional mean field particle interpretation of the flow of their final time marginals. In contrast to traditional genealogical tree based models, these new particle algorithms can be used to compute normalized additive functionals on-the-fly as well as their limiting occupation measures with a given precision degree that does not depend on the final time horizon.

We present uniform convergence results w.r.t. the time horizon parameter as well as functional central limit theorems and exponential concentration estimates, yielding what seems to be the first results of this type for this class of models. We also illustrate these results in the context of computational physics and financial mathematics.

9. Laurent Denis (University of Evry-Val-d'Essonne)

   The Lent Particle Method and its applications

   This talk is based on several joint works with N. Bouleau. We present a new approach to absolute continuity and regularity of laws of Poisson functionals. The theoretical framework is that of local Dirichlet forms. The method gives rise to a new explicit calculus that we first show on some simple examples : it consists in adding a particle and taking it back after computing the gradient. This method permits to to develop a Malliavin calculus on the Poisson space and to obtain in a simple way existence of density and regularity of laws of Poisson functionals such as Levy areas, solutions of SDE's driven by Poisson measure...

10. Nicole El Karoui (Université Paris VI)

    TBA

11. Caroline Hillairet (Ecole Polytechnique)

    Credit risk with asymmetric information on the default threshold

    This is a joint work with Ying Jiao. We consider a financial market with a default risk of a firm in a general barrier model : we model the default time has the first passage time of the firm value process under a random barrier L. We study the impact of asymmetric information on the barrier L in the pricing of credit derivatives and in the optimization of the expected utility from terminal wealth.

    The managers of the firm fix the barrier L d and thus have complete information on it, while the investors on the market only observe whether the default has occured or not. Different information structures are distinguished using the framework of enlargement of filtrations. We specify risk neutral probabilities and we evaluate default sensitive contingent claims in these cases. Finally, we study the managers' optimal investment strategy in a market with a stock exposed to a

counterparty risk, the default of the firm inducing a jump in the stock price.

12. Mingshang Hu (Shandong University)

    G-Levy processes under sublinear expectations

    We introduce G-Levy processes which develop the theory of processes with independent and stationary increments under the framework of sublinear expectations. We then obtain the Levy-Khintchine formula and the existence for G-Levy processes. Specially, we introduce G-Poisson processes.

13. Xiangdong Li (Institute of Applied mathematics, AMSS)

    Some Greeks formulas for complete and incomplete financial markets

    In this talk, we present some recent results on the study of Greeks formulas for complete and incomplete financial markets using the Stochastic Variational Calculus and the Stochastic Differential Geometry founded by Paul Malliavin.

14. Gechun Liang (Oxford University)

    Rollover Risk of Debt and Bank Run

    We present a new dynamic bank run model for liquidity risk where the financial institution finances by a mixture of short- and long-term debt. By this model, we incorporate rollover risk into structural credit risk models. We show the financial institution fails because of creditors' bank run rather than financial institution's insolvency. The problem is reduced to an optimal stopping time problem, which is solved by the approaches of reflected BSDE and free-boundary PDE

15. Chunhua Ma (Nankai University)

    Estimation in the Cox-Ingersoll-Ross driven by α-stable Lévy noises

    The classical Cox-Ingersoll-Ross (CIR) model has been used in the interest rate modeling. However, interest rates do not evolve continuously over time. In this talk, we consider the CIR model driven by spectrally positive α-stable Lévy noises ($1 < \alpha < 2$), which is also known as the sub-critical continuous state branching process with immigration given by the following SDE :

    $$dr(t) = (a - br(t))dt + c\sqrt[\alpha]{r(t-)}dZ(t),$$

    where $a > 0$, $b > 0$, $c > 0$, and $Z(\cdot)$ is a spectrally positive α-stable Lévy process. The exponentially ergodicity and extremal behavior of the above process are studied. Part of properties can also be extended to the case of general affine processes. Based on these properties, we obtain the asymptotic distributions for the (weighted) conditional least squares estimators of the drift parameters a and b.

16. Jin Ma (University of Southern California)

    Optimal Liquidation Problem with Dynamic Equilibrium Limit Order Book

    We study an optimal liquidation problem in an order-driven market, in which the shape of the limit order book (LOB) is given endogenously via a "principle of equilibrium". The resulting density function of the order book is nonlinear, random, and time inhomogeneous. The liquidation problem is formulated as a singular-type stochastic control problem with special constraints. We establish the Dynamic Programming Principle and prove that the value function is the viscosity solution to the corresponding Hamilton-Jacobi-Bellman equation. Assuming zero resilience, we analyze the optimal strategy in terms of the solution of the HJB equation, which takes the form that is not commonly seen in the singular control literature.

17. Anis Matoussi (Université du Maine)

    Quadratic BSDE's with jumps and unbounded terminal condition : semimartingale approach

18. Shige Peng (Shandong University)

    BSDE, PDE and Nonlinear Expectation

    A solution of a linear BSDE (Backward Stochastic Differential Equation) is a discounted martingale with a prescribed terminal value under a specific probability called martingale measure through a Girsanov transformation. In fact a general nonlinear BSDE, though very different from an SDE, can be still solved by a fixed point approach. It turns out that the BSDE can be considered as a nonlinear Girsanov transformation and that the solution of the BSDE is a nonlinear martingale under a nonlinear expectation, called g-expectation. The corresponding nonlinear Feynman-Kac formula tells us that, once the coefficients depend only on the state of the Brownian path, then the BSDE is a quasilinear PDE of parabolic type. This reveals that, a general (non-Markovian) BSDE is in fact a PDE in which the Brownian path plays the role of state variable x. The g-expectation then is the nonlinear semigroup associated with the PDE. The above Brownian motion can also be replaced by a general and possibly degenerate Markovian process. For a fully nonlinear parabolic PDE, can one still establish the corresponding BSDE, nonlinear expectation and path-dependence PDE ? The answer of this deep problem is : the Wiener probability measure corresponding to the Brownian motion can no longer be the reference probability space of the corresponding nonlinear expectation since the later is fully nonlinear which cannot be absolutely continuous w. r. t. this measure. A direct solution to this problem is to use the PDE to construct the corresponding nonlinear expectation, called G-expectation. The nonlinear martingale under this

expectation, called G-martingale, is the solution of the BSDE which can be regarded as the corresponding fully nonlinear path-dependence PDE. It turns out that a welldesigned G-expectation can be used to dominate a large set of fully nonlinear expectations, or BSDE. In this framework, the corresponding canonic process is called G-Brownian motion which is a continuous process with independent and stable increments. The increments are proved to be G-normal distributed which coincides with the limit distribution of the central limit theorem under a nonlinear expectation. This approach also gives us a pedagogically direct and simple access to the theory of G-expectation, G-Brownian motion and the corresponding stochastic calculus of Itôs type.

19. Monique Pontier (Université de Toulouse)

    Optimal Capital Structure with Stochastic Volatility

    This is a joint work with Flavia Barsotti. We analyze the capital structure of a firm in a infinite time horizon framework following Lelands idea [2] under the more general hypothesis that the firm value process belongs to a fairly large class of stochastic volatility models. In such a scheme, we describe and analyze the effects of stochastic volatility on all variables describing the capital structure. The endogenous failure level is derived in order to exploit the optimal amount of debt chosen by the firm. Exploiting optimal capital structure we find that the stochastic volatility framework seems to be a robust way to improve results in the direction of both higher spreads and lower leverage ratios in a quantitatively significant way. The used method to approximate stochastic volatility is the one defined in [1].

    References

    [1] FOUQUE J.P., PAPANICOLAOU G., RONNIE K.R., Derivatives in Financial Markets with Stochastic Volatility, Cambridge University Press (2000).

    [2] LELAND H.E. (1994), Corporate debt value, bond covenant, and optimal capital struc- ture, The Journal of Finance, 49, 1213-1252.

20. Mathieu Rosenbaum (Ecole Polytechnique)

    Optimal discretization of hedging strategies with jumps

    In this work, we consider the hedging error due discrete trading of strategies with jumps. We propose a framework enabling to (asymptotically) optimize the discretization times. More precisely, a strategy is said to be optimal if for a given cost function, no strategy has (asymptotically) a lower mean square error for a smaller cost. We focus on strategies based on hitting times and give explicit expressions for the optimal strategies. This is joint work with Peter Tankov.

21. Simone Scotti (Université Paris Diderot — Paris 7)

    Bid-Ask Spread Modelling, a Perturbation Approach

Our objective is to study liquidity risk, in particular the so-called "bid-ask spread", as a by-product of market uncertainties. "Bid-ask spread", and more generally "limit order books" describe the existence of different sell and buy prices, which we explain by using different risk aversions of market participants. The risky asset follows a diffusion process governed by a Brownian motion which is uncertain. We use the error theory with Dirichlet forms to formalize the notion of uncertainty on the Brownian motion. This uncertainty generates noises on the trajectories of the underlying asset and we use these noises to expound the presence of bid-ask spreads. In addition, we prove that these noises also have direct impacts on the mid-price of the risky asset. We further enrich our studies with the resolution of an optimal liquidation problem under these liquidity uncertainties and market impacts. To complete our analysis, some numerical results will be provided.

22. Shiqi Song (Université d'Evry)

Martingale representation property in progressively enlarged filtrations

This is a joint work with Monique Jeanblanc. In mathematical modelling of financial market, the martingale representation property (in abbreviation $\mathfrak{M}$rp) plays an essential role in the analysis and resolution of problems. On the other hand, in recent years, the theory of progressive enlargement of filtration settles itself more and more in the credit market modelling. Our work is at the junction of the latter with the former.

Let $(\Omega, \mathcal{A}, \mathbb{F}, \mathbb{P})$ be a filtered probability space and let τ be a positive random variable on $(\Omega, \mathcal{A})$. The progressively enlarged filtration relative to $\mathbb{F}$ with τ, is defined as $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$ where $\mathcal{G}_t = \cap_{s>t}(\mathcal{F}_s \vee \sigma(\tau \wedge s))$ for $t \geq 0$. Suppose that $\mathfrak{M}$rp holds in $\mathbb{F}$. Introducing the notion of H-measure, we prove a sufficient condition for $\mathfrak{M}$rp to hold in $\mathbb{G}$. Our result is efficient enough to cover the known situations such as Barlow's theorem in the case of honest time, Kusuoka's result in the case of immersion property, the Cox model, the density hypothesis, etc. However, the main application of our result lies in the following model established in one of our preceding papers.

Concretely, we have been gived a $(\mathbb{P}, \mathbb{F})$ supermartingale $Ne^{-\Lambda}$, where Λ is an $\mathbb{F}$-adapted continuous increasing process and N is a cdlg positive $(\mathbb{P}, \mathbb{F})$ local martingale, such that $\Lambda_0 = 0$ and $0 \leq N_t e^{-\Lambda_t} < 1$ for all $0 \leq t < \infty$. Assuming that any $(\mathbb{P}, \mathbb{F})$ local martingale are continuous, we have established that, for any $(\mathbb{P}, \mathbb{F})$ local martingale Y and for any bounded differentiable function f with bounded continuous derivative and $f(0) = 0$, there exists (on some extension of $(\Omega, \mathbb{F}, \mathbb{P})$) a random time τ, such that, for any $u > 0$, the $(\mathbb{P}, \mathbb{F})$ martingale $M^u = (M_t^u)_{t\geq u}$, where $M_t^u = \mathbb{P}[\tau \leq u | \mathcal{F}_t]$ for $t \geq u$, satisfies the

stochastic differential equation :

$$(\natural_u)\begin{cases} dX_t = X_t\left(-\frac{e^{-\Lambda_t}}{1-Z_t}dN_t + f(X_t-(1-Z_t))dY_t\right),\ u \le t < \infty \\ X_u = 1 - N_u e^{-\Lambda_u} \end{cases}$$

The conditional survival probability associated with such a τ is simply $Ne^{-\Lambda}$ and, thanks to the equation($\natural$), τ possesses good properties for modelling the credit market after a default. The question was if such a model possesses $\mathfrak{M}$rp. The present work gives an answer.

Let us make a point at the technical aspect of the problem. It is easy to see the difficulty to prove $\mathfrak{M}$rp in general. The elaborated proof of Barlow's theorem in Jeulin's book is of a good example. Looking for efficient method to deal with general enlargement of filtration is still an actual topic nowaday. Our work offers an interesting option to this technical problem.

23. Nizar Touzi (Ecole Polytechnique)

Model independent bounds under calibration constraints : a stochastic control approach

We develop a stochastic control approach for the derivation of model independent bounds for derivatives under various calibration constraints. Unlike the previous literature, our formulation seeks the optimal no arbitrage bounds given the knowledge of the distribution at some (or various) point in time. By convex duality techniques, this problem is converted into an optimal transportation problem along controlled stochastic dynamics. We also provide precise connections with the Azema-Yor solution of the Skorohod Embedding problem, and we obtain some extensions.

24. Dewen Xiong (Jiaotong University)

Modeling the forward CDS spreads with jumps

In this paper, we consider the forward CDS in the frame of stochastic interest rate whose term structure is modeled by HJM with jumps adapted to the filtration $\mathbb{F}$(see in [Bjoerk-Kabanov-Runggaldier1997] or in [Xiong-Kohlmann2010b]). Under the assumption that the density process of the default is a bounded $\mathbb{F}$-predictable process, we obtain a quadratic-exponential type BSDEs system similar to [Xiong-Kohlmann2010b] which always has a unique solution (X, θ, ϑ). By the solution of such BSDEs system, we can describe the dynamic of the the pre-default values of the defaultable bond, the defaultable forward Libor rates and the restricted defaultable forward measure (see in [Eberlein-Kluge-Schoenbucher-2006]) explicitly. Then we introduce another quadratic-exponential type BSDEs system (called **adjoint BSDEs system**) which also always has a unique solution, and by the solution, we describe the dynamic of the fair spread of the forward

CDS with the tenor structure $\mathbb{T} = \{a = T_0 < T_1 < \cdots < T_n = b\}$ explicitly.

25. Zuoquan Xu (Hong Kong polytechnic university)
Time-inconsistent decision-making problem
Buying and selling strategy are fundamentally important in security trading problems. Several typical trading strategies such as Buy-and-hold, cut-loss-and-take-profit, cut-loss-and-let-profit-run, are observed in practice. However, there is no single existing traditional optimal trading model could derive all of the above trading strategies at once. At the same time, we can also observe that people tend to use pre-committed strategies at initial and will change their mind later in practice, in another words, their trading strategies are time-inconsistent. We will formulate an optimal stopping model by incorporating probability distortion to derive the above mentioned strategies at once and give an explanation why people use those time-inconsistent strategies.
26. Phillip Yam (Chinese University of Hong Kong)
Linear Quadratic Mean Field Games
The theory of Mean Field Games has grown rapidly after the pioneering paper by Lasry and Lions (2007). For the recent development and its applications, one can refer to, for example, the survey (Gueant et al. 2011) and the references therein. In this talk, I shall introduce a class of Mean Field Games in which both the pay-off function and cost functional are quadratic in state variable, control variable together with the mean field term ; besides, the controlled dynamics is linear and also consists of a mean field term. We shall also briefly discuss about the existence and uniqueness of both the value function and the optimal control of each of these Mean Field Games ; indeed, we can establish them by using a method that combines adjoint equation approach and the theory of backward stochastic differential equations.
27. Zhongxing Ye (Jiaotong University)
Some new results on pricing credit derevatives based on intensity model with interest rate risk and courterparty risk
We mainly study the pricing formulas of credit default swap (CDS) in intensity-based models with counterparty risk. We assume that the default intensity of firm depends on the stochastic interest rate driven by the jump-diffusion process and the default states of counterparty firms. Moreover, we make use of the hyperbolic function to illustrate the attenuation effect of correlated defaults between counterparties. Our models are extensions of the models in Jarrow and Yu (2001) and Bai, Hu and Ye (2007). Furthermore, we make use of the techniques in Park (2008) to compute the conditional distribution of default times and present the explicit prices of bond and CDS in the primary-secondary and looping default frameworks.

28. Qi Zhang (Fudan University)

On the Cauchy Problem for Degenerate Backward Stochastic PDEs in Sobolev Spaces

We consider the Cauchy problem of linear degenerate backward stochastic partial differential equations and obtain the existence and uniqueness results in Sobolev space $L^p(\Omega; C([0,T]; W^{m,p}))$ where $m \geq 1$ and $p \geq 2$. To illustrate the application, we give a maximum principle for optimal control of degenerate stochastic PDEs.

29. Xunyu Zhou (Chinese university of Hong Kong and Oxford university)

Hope, Fear and Aspiration

We propose a new portfolio choice model in continuous time which features three key human emotions in choice-making : hope, fear and aspiration. By applying recently developed quantile formulation, we solve this model completely. Fear and hope indices are proposed via the curvatures of probability distortion functions to study the impact of these emotions on investment behaviors. This is a joint work with Xuedong He.