中国科学院数学与系统科学研究院应用数学所

偏微分方程及其应用中心

学术报告

报告题目： **Endpoint regularity of maximal functions**

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时　　间： **2024 年 6 月 12 日（星期三）15:00-16:00**

地　　点： **数学院南楼 N613**

摘　　要： Given an integrable real valued function f on ℝ^d, its Hardy-Littlewood maximal function Mf maps a point x in ℝ^d to the largest average value that can be achieved by averaging |f| over any ball centered in x. The classical Hardy-Littlewood maximal function theorem states that this maximal operator is a bounded operator on the Lebesgue space L^p(ℝ^d) if and only if 1 < p <= ∞. Indeed, for every nonzero function f in L^1(ℝ^d) its maximal function Mf is not even in L^1(ℝ^d).

In 1997 Juha Kinnunen proved the corresponding result for the gradient of the maximal function, i.e. that the L^p(ℝ^d)-norm of the gradient of the maximal function is controlled by the L^p(ℝ^d)-norm of the gradient of the function if 1 < p <= ∞. However, he provides no counterexample in the endpoint, leaving open the possibility that the gradient bound in fact holds also for p = 1.

In 2004 Hajłasz and Onninen formally posed the question if the Hardy-Littlewood maximal operator on ℝ^d satisfies the endpoint gradient bound. It has since attracted considerable attention, motivated by to the elementary nature of the maximal operator and by the relative simplicity of the proof of the Hardy-Littlewood maximal function theorem and of Kinnunens 1997 result. Many special cases, generalizations and variations of this problem have been explored, with partial success. The original question by Hajłasz and Onninen remains unanswered.