中国科学院

随机复杂结构与数据科学重点实验室

学术报告

题　　目： A Hamilton-Jacobi theory to hydrodynamic limit of action minimizing collective dynamics, an overview

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时　　间： **2023 年 7 月 20 日（周四）　9:30-10:30**

地　　点： **数学院 N613**

摘　　要： We study hydrodynamic limit for interacting particles following deterministic Hamiltonian dynamics. Answers to such question can help us better understand mechanical nature of stochastic models in statistical and continuum mechanics. Traditional approach on such a program faces many difficulties. One of them is about rigorous justification of canonical type ensembles. This is because relevant deterministic ergodic theory is still largely out of reach. Another huge barrier is on making sense of rigorous meaning of hyperbolic conservation laws. Such PDEs are needed to express F=ma and thermodynamic relations in the continuum.

To make possible progress, we insist to start with deterministic Hamiltonian models as the building blocks. But we allow reduced expectations on other aspects of the issue, so that we can introduce mathematical modifications to simplify. To be precise, first, we only study (global in time) action minimizing (instead of critical point) dynamics of particles. Second, we only consider situations where particle interactions are weak enough so that thermodynamics is very simple (isentropic case). Third, we

are content with not seeing probability models explicitly in the end at this point, but only deterministic variational problems matching asymptotic (e.g. large deviations) of the probability models with other hidden scaling parameters. Fourth, we only study the hydrodynamic limit from a variational point of view, not its evolutionary equation form as usually done in the PDE community. Hence avoid making precise sense of F=ma.

With the above reductions, we then explore a new formulation of the hydrodynamic limit issue as abstract multi-scale Hamilton-Jacobi theory in space of probability measures. Interestingly, there are enough mathematical tools available for us to make meaningful results. We give an overview on how this can be done.

This is work in progress with Toshio Mikami in Tsuda University, Japan.