

成果一.

董昭研究员与合作者的论文 SHARP NONUNIQUENESS OF SOLUTIONS TO STOCHASTIC NAVIER-STOKES EQUATIONS 被 SIAM JOURNAL ON MATHEMATICAL ANALYSIS 接收发表。

摘要: In this paper we establish a sharp nonuniqueness result for stochastic d -dimensional ($d \geq 2$) incompressible Navier-Stokes equations. First, for every divergence-free initial condition in L^2 we show existence of infinitely many global-in-time probabilistically strong and analytically weak solutions in the class $L^\alpha(\Omega, (L^t L^\infty) - L^p)$ for any $1 \leq p < 2$, $\alpha \geq 1$. Second, we prove that the above result is sharp in the sense that pathwise uniqueness holds in the class of $L^t(p) L^q$ for some p is an element of $[2, \infty]$, q is an element of $(2, \infty]$ such that $2/p + d/q \leq 1$, which is a stochastic version of Ladyzhenskaya-Prodi-Serrin criteria. Moreover, for the stochastic d -dimensional incompressible Euler equation, the existence of infinitely many global-in-time probabilistically strong and analytically weak solutions is obtained. Compared to the stopping time argument used in Hoffmanov & aacute;, Zhu, and Zhu [J. Eur. Math. Soc. (JEMS), to appear; Ann. Probab., 51 (2023), pp. 524-579], we developed a new stochastic version of the convex integration. More precisely, we introduce expectation during convex integration scheme and construct directly solutions on the whole time interval $[0, \infty)$.

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成果二.

丁超副研究员与合作者的论文 STRONG VARIATIONAL SUFFICIENCY FOR NONLINEAR SEMIDEFINITE PROGRAMMING AND ITS IMPLICATIONS 被 SIAM JOURNAL ON OPTIMIZATION 接收发表。

摘要: Strong variational sufficiency is a newly proposed property, which turns out to be of great use in the convergence analysis of multiplier methods. However, what this property implies for nonpolyhedral problems remains a puzzle. In this paper, we prove the equivalence between the strong variational sufficiency and the strong second-order sufficient condition (SOSC) for nonlinear semidefinite programming (NLSDP) without requiring the uniqueness of the multiplier or any other constraint qualifications. Based on this characterization, the local convergence property of the augmented Lagrangian method (ALM) for NLSDP can be established under the strong SOSC in the absence of constraint qualifications. Moreover, under the strong SOSC, we can apply the semismooth Newton method to solve the ALM subproblems of NLSDP because the positive definiteness of the generalized Hessian of augmented Lagrangian function is satisfied.

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成果三.

黄飞敏研究员与合作者的论文 ISOTHERMAL LIMIT OF ENTROPY SOLUTIONS OF THE EULER EQUATIONS FOR ISENTROPIC GAS DYNAMICS 被 SIAM JOURNAL ON MATHEMATICAL ANALYSIS 接收发表。

摘要: We are concerned with the isothermal limit of entropy solutions in L^∞ , containing the vacuum states, of the Euler equations for isentropic gas dynamics. We prove that the entropy solutions in L^∞ of the isentropic Euler equations converge strongly to the corresponding entropy solutions of the isothermal Euler equations, when the adiabatic exponent $\gamma > 1$. This is achieved by combining careful entropy analysis and refined kinetic formulation with a compensated compactness argument to obtain the required uniform estimates for the limit. The entropy analysis involves careful estimates for the relation between the corresponding entropy pairs for the isentropic and isothermal Euler equations when the adiabatic exponent $\gamma > 1$. The kinetic formulation for the entropy solutions of the isentropic Euler equations with the uniformly bounded initial data is refined, so that the total variation of the dissipation measures in the formulation is locally uniformly bounded with respect to $\gamma > 1$. The explicit asymptotic analysis of the Riemann solutions containing the vacuum states is also presented.

论文链接: <http://dx.doi.org/10.1137/23M1549948>

成果四.

朱湘禅研究员与合作者的论文 SINGULAR KINETIC EQUATIONS AND APPLICATIONS 被 ANNALS OF PROBABILITY 接收发表。

摘要: In this paper we study singular kinetic equations on \mathbb{R}^2 by the paracontrolled distribution method introduced in Gubinelli, Imkeller and Perkowski (Forum Math. Pi 3 (2015) e6-75). We first develop paracontrolled calculus in the kinetic setting and use it to establish the global well-posedness for the linear singular kinetic equations under the assumptions that the products of singular terms are well defined. We also demonstrate how the required products can be defined in the case that singular term is a Gaussian random field by probabilistic calculation. Interestingly, although the terms in the zeroth Wiener chaos of regularization approximation are not zero, they converge in suitable weighted Besov spaces, and no renormalization is required. As applications the global well-posedness for a nonlinear kinetic equation with singular coefficients is obtained by the entropy method. Moreover, we also solve the martingale problem for nonlinear kinetic distribution dependent stochastic differential equations with singular drifts.

论文链接: <http://dx.doi.org/10.1214/23-AOP1666>

成果五.

黄飞敏研究员与合作者的论文 Global Finite-Energy Solutions of the Compressible Euler-Poisson Equations for General Pressure Laws with Large Initial Data of Spherical Symmetry 被 COMMUNICATIONS IN MATHEMATICAL PHYSICS 接收发表。

摘要:

Abstract: We are concerned with global finite-energy solutions of the three-dimensional compressible Euler–Poisson equations with *gravitational potential* and *general pressure law*, especially including the constitutive equation of *white dwarf stars*. In this paper, we construct global finite-energy solutions of the Cauchy problem for the Euler–Poisson equations with large initial data of spherical symmetry as the inviscid limit of the solutions of the corresponding Cauchy problem for the compressible Navier–Stokes–Poisson equations. The strong convergence of the vanishing viscosity solutions is achieved through entropy analysis, uniform estimates in L^p , and a more general compensated compactness framework via several new ingredients. A key estimate is first established for the integrability of the density over unbounded domains independent of the vanishing viscosity coefficient. Then a special entropy pair is carefully designed via solving a Goursat problem for the entropy equation such that a higher integrability of the velocity is established, which is a crucial step. Moreover, the weak entropy kernel for the general pressure law and its fractional derivatives of the required order near vacuum ($\rho = 0$) and far-field ($\rho = \infty$) are carefully analyzed. Owing to the generality of the pressure law, only the $W_{\text{loc}}^{-1,p}$ -compactness of weak entropy dissipation measures with $p \in [1, 2)$ can be obtained; this is rescued by the equi-integrability of weak entropy pairs which can be established by the estimates obtained above, so that the div-curl lemma still applies. Finally, based on the above analysis of weak entropy pairs, the L^p compensated compactness framework for the compressible Euler equations with general pressure law is established. This new compensated compactness framework and the techniques developed in this paper should be useful for solving further nonlinear problems with similar features.

论文链接: <http://dx.doi.org/10.1007/s00220-023-04916-1>

成果六.

朱湘禅研究员与合作者的论文 Nonuniqueness in law of stochastic 3D Navier-Stokes equations 被 JOURNAL OF THE EUROPEAN MATHEMATICAL SOCIETY 接收发表。

摘要: We consider the stochastic Navier-Stokes equations in three dimensions and prove that the law of analytically weak solutions is not unique. In particular, we focus on three examples of a stochastic perturbation: an additive, a linear multiplicative and a nonlinear noise of cylindrical type, all driven by a Wiener process. In these settings, we develop a stochastic counterpart of the convex integration method introduced recently by Buckmaster and Vicol. This permits us to construct probabilistically strong and analytically weak solutions defined up to a suitable stopping time. In addition, these solutions fail to satisfy the corresponding energy inequality at a prescribed time with a prescribed probability. Then we introduce a general probabilistic construction used to extend the convex integration solutions beyond the stopping time and in particular to the whole time interval $[0, \infty)$. Finally, we show that their law is distinct from the law of solutions obtained by Galerkin approximation. In particular, nonuniqueness in law holds on an arbitrary time interval $[0, T]$, $T > 0$.

论文链接: <http://dx.doi.org/10.4171/JEMS/1360>

成果七.

张世华研究员与合作者的论文 High-density generation of spatial transcriptomics with STAGE 被 NUCLEIC ACIDS RESEARCH 接收发表。

摘要: Spatial transcriptome technologies have enabled the measurement of gene expression while maintaining spatial location information for deciphering the spatial heterogeneity of biological tissues. However, they were heavily limited by the sparse spatial resolution and low data quality. To this end, we develop a spatial location-supervised auto-encoder generator STAGE for generating high-density spatial transcriptomics (ST). STAGE takes advantage of the customized supervised auto-encoder to learn continuous patterns of gene expression in space and generate high-resolution expressions for given spatial coordinates. STAGE can improve the low quality of spatial transcriptome data and smooth the generated manifold of gene expression through the de-noising function on the latent codes of the auto-encoder. Applications to four ST datasets, STAGE has shown better recovery performance for down-sampled data than existing methods, revealed significant tissue structure specificity, and enabled robust identification of spatially informative genes and patterns. In addition, STAGE can be extended to three-dimensional (3D) stacked ST data for generating gene expression at any position between consecutive sections for shaping high-density 3D ST configuration. Graphical Abstract

论文链接: <http://dx.doi.org/10.1093/nar/gkae294>

成果八.

张羊晶助理研究员与合作者的论文 On Efficient and Scalable Computation of the Nonparametric Maximum Likelihood Estimator in Mixture Models 被 JOURNAL OF MACHINE LEARNING RESEARCH 接收发表。

摘要: In this paper, we focus on the computation of the nonparametric maximum likelihood estimator (NPMLE) in multivariate mixture models. Our approach discretizes this infinite dimensional convex optimization problem by setting fixed support points for the NPMLE and optimizing over the mixing proportions. We propose an efficient and scalable semismooth Newton based augmented Lagrangian method (ALM). Our algorithm outperforms the state-of-the-art methods (Kim et al., 2020; Koenker and Gu, 2017), capable of handling $n \geq 10^6$ data points with $m \geq 10^4$ support points. A key advantage of our approach is its strategic utilization of the solution's sparsity, leading to structured sparsity in Hessian computations. As a result, our algorithm demonstrates better scaling in terms of m when compared to the mixsqp method (Kim et al., 2020). The computed NPMLE can be directly applied to denoising the observations in the framework of empirical Bayes. We propose new denoising estimands in this context along with their consistent estimates. Extensive numerical experiments are conducted to illustrate the efficiency of our ALM. In particular, we

employ our method to analyze two astronomy data sets: (i) Gaia-TGAS Catalog (Anderson et al., 2018) containing approximately 1.4×10^6 data points in two dimensions, and (ii) a data set from the APOGEE survey (Majewski et al., 2017) with approximately 2.7×10^4 data points.

论文链接: jmlr.org/papers/volume25/22-1120/22-1120.pdf

成果九.

骆顺龙研究员与合作者的论文 Equioverlapping measurements as extensions of symmetric informationally complete positive operator valued measures 被 PHYSICAL REVIEW A 接收发表。

摘要: Equioverlapping measurements, as a natural class of quantum measurements with the appealing property of equal overlap between any pair of measurement operators, generalize both the notions of equiangular tight frames (including von Neumann measurements) and symmetric informationally complete positive operator valued measures (SIC-POVMs). The structures of equioverlapping measurements in general dimensions are rather subtle and complicated. In this work, we reveal some structural properties of equioverlapping measurements which may be useful for constructing and classifying equioverlapping measurements. In particular, we obtain two bounds for the number of measurement operators in an equioverlapping measurement. We further illustrate how equioverlapping measurements go beyond SIC-POVMs in a nontrivial fashion with some illuminating examples in lower dimensions (two, three, and four). Finally, we present the challenging problem of fully classifying equioverlapping measurements and discuss some related perspectives.

论文链接: <http://dx.doi.org/10.1103/PhysRevA.109.012218>

成果十.

张世华研究员与合作者的论文 STAMarker: determining spatial domain-specific variable genes with saliency maps in deep learning 被 NUCLEIC ACIDS RESEARCH 接收发表。

摘要: Spatial transcriptomics characterizes gene expression profiles while retaining the information of the spatial context, providing an unprecedented opportunity to understand cellular systems. One of the essential tasks in such data analysis is to determine spatially variable genes (SVGs), which demonstrate spatial expression patterns. Existing methods only consider genes individually and fail to model the interdependence of genes. To this end, we present an analytic tool STAMarker for robustly determining spatial domain-specific SVGs with saliency maps in deep learning. STAMarker is a three-stage ensemble framework consisting of graph-attention autoencoders, multilayer perceptron (MLP) classifiers, and saliency map computation by the backpropagated gradient. We illustrate the effectiveness of STAMarker and compare it with several commonly used competing methods on various spatial transcriptomic data generated by different platforms. STAMarker considers all genes at

once and is more robust when the dataset is very sparse. STAMarker could identify spatial domain-specific SVGs for characterizing spatial domains and enable in-depth analysis of the region of interest in the tissue section.

论文链接: <http://dx.doi.org/10.1093/nar/gkad801>